

## **Harmonic Progression**

A Harmonic Progression (HP) is defined as a sequence of real numbers which is determined by taking the reciprocals of the arithmetic progression that does not contain 0. In the harmonic progression, any term in the sequence is considered as the harmonic means of its two neighbours. For example, the sequence  $a, b, c, d, \dots$  is considered as an arithmetic progression, the harmonic progression can be calculated as  $1/a, 1/b, 1/c, 1/d, \dots$

**Harmonic Mean:** Harmonic mean is calculated as the reciprocal of the arithmetic mean of the reciprocals. The formula to calculate the harmonic mean is given by:

$$\text{Harmonic Mean} = n / [(1/a) + (1/b) + (1/c) + (1/d) + \dots]$$

Where,

$a, b, c, d$  are the values and  $n$  is the number of values present.

## **Harmonic Progression Formula**

To solve the harmonic progression problems, we should find the corresponding arithmetic progression sum. It means that the  $n$ th term of the harmonic progression is equal to the reciprocal of the  $n$ th term of the corresponding A.P. Thus the formula to find the  $n$ th term of the harmonic progression series is given as:

$$\text{The } n\text{th term of the Harmonic Progression (H.P)} = 1 / [a + (n-1)d]$$

Where

“ $a$ ” is the first term of A.P

“ $d$ ” is the common difference

“ $n$ ” is the number of terms in A.P

The H.P formula is also written as:

$$\text{The } n\text{th term of H.P} = 1 / (\text{nth term of the corresponding A.P})$$

## **Harmonic Progression Sum**

If  $1/a, 1/a+d, 1/a+2d, \dots, 1/a+(n-1)d$  is given harmonic progression, the formula to find the sum of  $n$  terms in the harmonic progression is given by the formula:

$$\text{Sum of } n \text{ terms, } S_n = \frac{1}{d} \ln \left\{ \frac{2a + (2n-1)d}{2a-d} \right\}$$

Where,

“ $a$ ” is the first term of A.P

“d” is the common difference of A.P

“ln” is the natural logarithm

### **Relation Between AP, GP and HP**

For any two numbers, if A, G, H are the Arithmetic, Geometric, and Harmonic Mean respectively, then the relationship between these three are given by:

- $G.M^2 = A.M \times H.M$ , where A.M, G.M, H.M are in G.P
- $A.M \geq G.M \geq H.M$